Laser Ignition of an Isentropically Compressed Dense *Z*-Pinch

F. Winterberg
Department of Physics, University of Nevada, Reno, U.S.A.
Reprint requests to Prof. F. W.; Fax: (775) 784-1398

Z. Naturforsch. **54a**, 459–464 (1999); received March 9, 1999

A dense z-pinch generated by a high voltage discharge over a corrugated helical sawtooth-shaped capillary tube with a solid DT core, is by shear flow stabilized against the $m=0$ and $m=1$ magnetohydrodynamic instabilities, and by rotational flow against the Rayleigh-Taylor instability. The shear- and rotational flow result from jet formation by the corrugated surface. A programmed voltage pulse can then isentropically compress the DT core to high densities, and if ignited at one end by a petawatt laser pulse, a thermonuclear detonation wave can be launched propagating along the z-pinch channel. The proposed z-pinch burn should also work without tritium as a thermonuclear detonation wave in deuterium.

1. Introduction

In two previous publications by the author [1, 2], it was proposed to combine the fast ignitor concept [3] with electric pulse power driven z-pinch implosions. The high densities needed for the fast ignitor concept were to be reached by radiative collapse for currents larger than the Pease-Braginskii current, and the stabilization of the pinch discharge by a superimposed axial shear flow generated with an auxiliary discharge.

Here we propose to combine the fast ignitor concept with isentropic electric pulse power compression. For this we consider solid DT inside a capillary tube which possesses a helical sawtooth corrugated surface (see Fig. 1). A pinch discharge along the surface of the tube produces jets emitted from the inner corner of the wedges, with the jets supersonically propagating along the tube surface stabilizing it against the $m = 0$ and $m = 1$ magnetohydrodynamic instabilities. In addition, the helical winding generates a rotational flow suppressing the Rayleigh-Taylor instability. To reach the highest possible densities the compression must be isentropic, requiring a current pulse with a properly chosen time dependence [4]. After the capillary tube is compressed, with the DT reaching its highest density, the DT is ignited with a laser pulse at one point launching a thermonuclear detonation wave from this point.

As it was emphasized [1, 2], a large amount of energy supplied by inexpensive electric pulse power doing the compression, and a small amount of energy supplied by expensive laser power doing the ignition, leads to a large thermonuclear gain with the least expense.

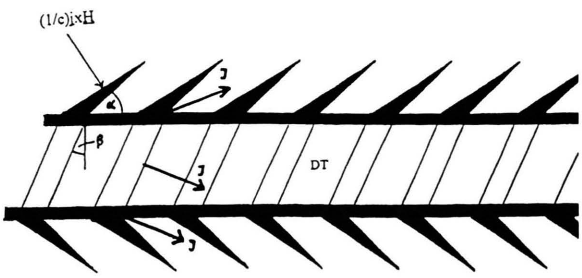

Fig. 1. Corrugated capillary tube filled with solid DT: α wedge angle, β pitch angle of corrugated surface, J jet, $(1/c)\boldsymbol{j}\times\boldsymbol{H}$ magnetic body force.

2. Shear Flow Stabilization

If the electrical conductivity of the capillary tube is sufficiently high, and the implosion short compared to the time needed for the magnetic field to diffuse into the tube, the magnetic field cannot penetrate into the tube. With the electric current $\boldsymbol{j}$ flowing along the surface of the capillary tube, the magnetic body force $(1/c)\boldsymbol{j}\times\boldsymbol{H}$ is directed perpendicularly onto the surface of the sawtooth (see Fig. 1). The magnetic field then acts like a piston onto the surface of the capillary tube, and one has to set the wedge implosion velocity v_0 equal the Alfvén velocity $v_A = H/\sqrt{4\pi\rho}$, where $H = 0.2\ I/r_0$ is the magnetic field of the current I in Ampere, with r_0 the initial radius of the capillary tube and ρ its density. As a result, sheetlike jets are ejected along the implod-

0932-0784 / 99 / 0800-0459 $ 06.00 © Verlag der Zeitschrift für Naturforschung, Tübingen · www.znaturforsch.com

ing sawtooth. The recoil from these jets generates a massive "slug" moving in a direction opposite to the jet. Both the jet and slug generate shear and implode the capillary tube.

According to G. Birkhoff et al. [5], the velocity of jet and slug are

$$v_j = (v_0/\sin\alpha)\,(1 + \cos\alpha),$$
$$v_s = (v_0/\sin\alpha)\,(1 - \cos\alpha), \quad (1)$$

where α is the sawtooth angle as shown in Figure 1.

The relative fractions of the jet – and slug mass are

$$m_j/m = (1/2)\,(1 - \cos\alpha),$$
$$m_s/m = (1/2)\,(1 + \cos\alpha). \quad (2)$$

The stagnation pressure of the jet and slug are

$$p_j = (1/4)\,(1 - \cos\alpha)\,\rho v_j^2 = (1/4)\,\rho v_A^2\,(1 + \cos\alpha),$$
$$p_s = (1/4)\,(1 + \cos\alpha)\,\rho v_s^2 = (1/4)\,\rho v_A^2\,(1 - \cos\alpha) \quad (3)$$

with the sum

$$p_j + p_s = (1/2)\,\rho v_A^2 = \mathrm{H}^2/8\pi \quad (4)$$

a necessary condition for shear flow stabilization.

3. Isentropic Compression

The implosion of the wedges is followed by the isentropic compression (with a superimposed shear and rotational motion) of the capillary tube and thermonuclear fuel. Isentropic compression by the pinch discharge requires a time dependence of the current given by [4]

$$I/I_0 = [1 - (t/t_0)^2]^{-2/5}. \quad (5)$$

If, for example, $\mathrm{I} = 10^7$ A, and the final capillary tube radius is $r = 5\times10^{-3}$ cm, the magnetic field there reaches $\mathrm{H} = 0.2\ I/r = 4\times10^8$ G, with a magnetic pressure $H^2/8\pi \sim 6\times10^{15}$ dyn/cm^2. At this pressure, the particle number density of isentropically compressed cold hydrogen would be $n \sim 100\,n_0$, where $n_0 = 5\times10^{22}$ cm^{-3} is the particle number density of solid hydrogen. Linear momentum conservation requires that in the course of the compression the axial flow velocity v_z remains unchanged.

The momentum density of the slug which gives the capillary tube its spin, but also its shear flow and radial implosion velocity, is obtained from (1) and (2) as

$$I_s = (\rho/2)\,v_A \sin\alpha. \quad (6)$$

It is equal and opposite to the momentum density of the ablated material. The tangential and radial components of (6) are

$$I_s \cos\alpha = (\rho/4)\,v_A \sin 2\alpha,$$
$$I_s \sin\alpha = (\rho/2)\,v_A \sin^2\alpha, \quad (7)$$

with a tangential and radial velocity component

$$v_t = (1/4)\,v_A \sin 2\alpha,$$
$$v_r = (1/2)\,v_A \sin^2\alpha. \quad (8)$$

The tangential velocity component has a maximum for $\alpha = 45°$, where one has

$$v_t = (1/4)\,v_A,$$
$$v_r = (1/4)\,v_A. \quad (9)$$

The tangential velocity component can be further decomposed into an axial and azimuthal component, the first generating shear and the second rotation:

$$v_z = (1/4)\,v_A \cos\beta,$$
$$v_\varphi = (1/4)\,v_A \sin\beta. \quad (10)$$

4. Suppression of the Rayleigh-Taylor Instability

For spherical implosions the Rayleigh-Taylor instability poses a serious problem limiting the maximum attainable compression. For a cylindrical implosion the situation is much better, because there a superimposed rotational motion can suppress this instability. A cylindrical implosion, though making use of this effect, requires a cylinder long compared to its diameter, but this is just realized in a sufficiently long z-pinch. To suppress the Rayleigh-Taylor instability, the capillary tube must be brought into fast rotational motion. It is here generated by the helical winding of its corrugated surface with a still to be determined pitch angle β (see Figure 1). For the high pressures considered, the imploding tube can be considered liquid. One therefore has an imploding vortex [6]. This problem will be treated in the appendix, where also the stability will be discussed in greater detail.

If one is only interested in estimates, a greatly simplified treatment of this problem is possible [7]. It assumes that at the time $t = 0$ all of the energy is deposited in the imploding shell, will part of the energy going into the radial implosion and part of it onto the rotational motion. If r_0 is the initial and r_1 the final implosion radius, further $v_r^{(0)}$, $v_\varphi^{(0)}$ and $v_r^{(1)}$, $v_\varphi^{(1)}$, the initial and final velocity components, with $v_r^{(1)} = 0$, energy and angular momentum conservation require that

$$(v_\varphi^{(1)})^2 = (v_r^{(0)})^2 + (v_\varphi^{(0)})^2, \quad (11)$$

$$r_1\, v_\varphi^{(1)} = r_0\, v_\varphi^{(0)}. \quad (12)$$

From this one obtains

$$v_r^{(0)}/v_\varphi^{(0)} = [(r_0/r_1)^2 - 1]^{1/2} \quad (13)$$

or for $r_0 \gg r_1$

$$v_r^{(0)}/v_\varphi^{(0)} \approx r_0/r_1. \quad (13a)$$

The radial deceleration

$$a_1 = (v_r^{(0)})^2/r_0 \quad (14)$$

leads to Rayleigh-Taylor instability, but the centrifugal acceleration

$$a_2 \approx (v_\varphi^{(1)})^2/r_1 \quad (15)$$

is counteracting and thus stabilizing. Suppression of the Rayleigh-Taylor instability is realized if

$$a_2/a_1 \gg 1. \quad (16)$$

With the help of (12) and (13a) this can be written as

$$a_2/a_1 = r_0/r_1 \gg 1. \quad (17)$$

Inserting the expressions $v_r = v_r^{(0)} = (1/4)\, v_A$, $v_\varphi = v_\varphi^{(0)} = (1/4)\, v_A \sin\beta$ from (9) and (10) into (13a), one has

$$\sin\beta = r_1/r_0 \ll 1. \quad (18)$$

For a 100 fold compression $r_1/r_0 = 0.1$ and $\beta = 6°$.

5. Ignition

For the fast ignitor concept the laser intensity must be

$$I_L \approx 10^{11}\, \lambda^{-2}\ \mathrm{W/cm^2} \quad (19)$$

with the plasma density larger than the critical density $n_c \approx 10^{13}\, \lambda^{-2}\ \mathrm{cm^{-3}}$. For yellow light one has $\lambda \approx 6 \times 10^{-5}$ cm and hence $n_c = 3\times10^{21}\ \mathrm{cm^{-3}}$, well below a ~ 100 fold compression of solid DT, where $n \approx 5\times10^{24}\ \mathrm{cm^{-3}}$.

For a thermonuclear detonation wave propagating along the pinch discharge channel the current must be larger than [2]

$$I > I_0 = \frac{c^2}{Ze}\sqrt{\frac{M_f\, \varepsilon_0}{2}}\ [\mathrm{esu}], \quad (20)$$

where M_f is the mass, Ze the charge, and ε_0 the kinetic energy of the charged fusion products. For the α-particles of the DT fusion reaction one finds that $I > I_0 \approx 4\times10^{15}$ esu ~ 1.3×10^6 A, a condition well satisfied for $I = 10^7$ A. The ignition is done by a laser pulse creating a hot spot from which a thermonuclear detonation wave is launched. This requires that the energy

$$E_{ign} = 3nkT\pi r^2 \lambda_0 \quad (21)$$

is supplied in a time less than the hydrodynamic disassembly time, where r is the radius and λ_0 the height of a DT cylinder heated to the ignition temperature $T \sim 10^8$ K, with λ_0 set equal to the range of the α particles of the DT fusion reaction:

$$\lambda_0 = a\,(kT)^{3/2}/n, \quad (a \approx 2\times10^{34}\mathrm{cm^{-2}erg^{-3/2}}) \quad (22)$$

hence

$$E_{ign} \approx 3ar^2(kT)^{5/2}. \quad (23)$$

For the above example $r \approx 5\times10^{-3}$ cm, and $T \geq 10^8$ K one finds $E_{ign} \geq 10^{11}$ erg = 10 kJ. At $T \sim 10^8$ K and $n \approx 5\times10^{24}\ \mathrm{cm^{-3}}$, the range of the α-particles is $\lambda_0 \approx 6\times10^{-3}$ cm. The disassembly time is $\tau \sim \lambda_0/a_0$, where $a_0 \sim 10^8$ cm/s is the velocity of sound at $T \sim 10^8$ K. One finds that $\tau \sim 6\times10^{-11}$ sec. The power of the ignition pulse is $P \gtrsim E_{ign}/\tau \gtrsim 2\times10^{14}$ Watt, attainable with petawatt lasers [8]. The area onto which the laser light has to be focussed is of the order $A \sim r\lambda_0$; for the given example $A \sim 3\times10^{-5}\ \mathrm{cm^2}$. The power per area then is $P/A \sim 10^{19}\ \mathrm{W/cm^2}$, which must be compared with $I_L \sim P/A = 10^{19}\ \mathrm{W/cm^2}$, required for the fast ignitor of $\lambda \approx 10^{-4}$ cm.

A thermonuclear detonation wave is also possible in deuterium without tritium, because the burn with D of the T and ^{3}He DD reaction products significantly increases the reaction rate of a burn wave in deuterium [9]. By comparison, no deuterium burn is possible in otherwise magnetically confined plasmas.

Appendix

A. Imploding Vortex

If the imploding rotating tube is approximated by a frictionless fluid, with a pressure outside the tube, one has an imploding vortex with a core pressure equal the plasma pressure inside the core. In this case one obtains a simple analytical solution. As long as the plasma pressure is small compared with the stagnation pressure of the imploding vortex, one can set the core pressure equal to zero. This assumption is mostly valid. A further simplification is to assume that the vortex is incompressible.

Introducing cylindrical coordinates, r, φ, z, the inner and outer radius of the vortex is at $r = R_i$ and $r = R_a$, with a pressure p_a acting on the outer surface. Because of rotational symmetry one has $\partial/\partial\varphi = 0$, and for a long vortex, with a length large compared to its diameter, one can put $\partial/\partial z = 0$. At $t = 0$, one has for the inner and outer radius $r = R_0$ and R_1. Going into a reference system where $v_z = 0$, the Euler- and continuity equation (with ρ the density of the imploding vortex) are

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}, \tag{A.1}$$

$$\frac{\partial v_\varphi}{\partial t} + v_r \frac{\partial v_\varphi}{\partial r} + \frac{v_r v_\varphi}{r} = 0, \tag{A.2}$$

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0, \tag{A.3}$$

Putting

$$v_r r = F(t), \tag{A.4}$$

$$v_\varphi r = G(t), \tag{A.5}$$

we find by inserting (A.5) into (A.2)

$$\dot{G} = 0 \tag{A.6}$$

hence

$$G = \ell = \text{const.}, \tag{A.7}$$

describing conservation of angular momentum. From (A.4) and (A.7) one has

$$\frac{v_r}{v_\varphi} = \frac{F(t)}{\ell}, \tag{A.8}$$

or

$$\frac{dr}{r d\varphi} = \frac{F(t)}{\ell}, \tag{A.9}$$

which by integration yields

$$r = R_i\, e^{[F(t)/\ell]\varphi} \tag{A.10}$$

This shows that the streamlines of the imploding vortex are logarithmic spirals, with the angle of inclination a function of t. (In a reference system where $v_z \neq 0$ they are helical spirals)

From (A.4) one has

$$\frac{\partial v_r}{\partial t} = \frac{\dot{F}}{r}, \tag{A.11}$$

which upon substitution into (A.1) results in

$$\frac{\dot{F}}{r} + v_r \frac{\partial v_r}{\partial r} - \frac{\ell^2}{r^3} = -\frac{1}{\rho}\frac{\partial p}{\partial r}. \tag{A.12}$$

We assume that $R_a \gg R_i$, which implies that the implosion velocity at $r = R_a$ is small compared to the implosion velocity at $r = R_i$. Integrating (A.12) from $r = R_i$ where $v_r = V_r$ and $p = 0$, to $r = R_a$ where $p = p_a$ and $v_r = 0$, one then finds

$$\dot{F} \log\left(\frac{R_a}{R_i}\right) - \frac{1}{2} V_r^2 - \frac{\ell^2}{2}\left[\frac{1}{R_i^2} - \frac{1}{R_a^2}\right] = \frac{p_a}{\rho}; \tag{A.13}$$

with $F(t) = R_i V_i$ and $\dot{R}_i = V_r$ one has

$$\begin{aligned} \dot{F} &= \dot{R}_i V_r + R_i \dot{V}_r \\ &= V_r^2 + R_i \frac{dV_r}{dR_i} \dot{R}_i \\ &= V_r^2 + \frac{R_i}{2}\frac{dV_r^2}{dR_i}. \end{aligned} \tag{A.14}$$

Eliminating $F(t)$ from (A.13) and (A.14) results in

$$\frac{dV_r^2}{dR_i} + \frac{1}{R_i}\left[2 + \frac{1}{\log(R_i/R_a)}\right] V_r^2 + \frac{\ell^2}{R_i \log(R_i/R_a)} \cdot \left[\frac{1}{R_i^2} - \frac{1}{R_a^2}\right] - \frac{2 p_a}{\rho R_i \log(R_i/R_a)} = 0. \tag{A.15}$$

With $x = R_i/R_a$ and $V_r^2 = y$, (A.15) is a first order linear differential equation

$$y' + f_1(x)\, y + f_2(x) = 0, \tag{A.16}$$

where

$$f_1(x) = \frac{1}{x}\left(2 + \frac{1}{\log x}\right),$$
$$f_2(x) = \frac{\ell^2}{R_a^2}\frac{1}{x \log x}\left(\frac{1}{x^2} - 1\right) - \frac{2 p_a}{\rho x \log x} \tag{A.17}$$

with the solution

$$y = e^{-\int f_1 dx}\left[\text{const.} - \int f_2\, e^{\int f_1 dx}\right]. \tag{A.18}$$

With (A.17) one finds

$$y = \frac{1}{x^2 \log x} \cdot \left[\text{const.} - \frac{\ell^2}{R_a^2}\left(\log x - \frac{x^2}{2}\right) + \frac{p_a}{\rho} x^2\right], \tag{A.19}$$

hence

$$V_r^2 = -\left(\frac{R_a}{R_i}\right)^2 \frac{1}{\log(R_a/R_i)} \cdot \left[\text{const.} + \frac{\ell^2}{R_a^2}\log\left(\frac{R_a}{R_i}\right) + \left(\frac{p_a}{\rho} + \frac{\ell^2}{2R_a^2}\right)\left(\frac{R_i}{R_a}\right)^2\right]. \tag{A.20}$$

The constant of integration is determined by the initial condition at $t = 0$. If at $t = 0$, $V_r = V_r^{(0)}$ and $V_\varphi = V_\varphi^{(0)}$, with $\ell = R_0 V_\varphi^{(0)}$, one finds

$$V_r^2 = \left(\frac{R_0}{R_i}\right)^2 \left(\frac{R_a}{R_1}\right)^2 \frac{\log(R_1/R_0)}{\log(R_a/R_i)} \cdot \left[(V_r^{(0)})^2 + (V_\varphi^{(0)})^2\left(1 - \frac{\log(R_a/R_i)}{\log(R_1/R_0)}\left(\frac{R_1}{R_a}\right)^2\right) + \frac{1}{\log(R_1/R_0)}\left\{\left(\frac{p_a}{\rho} + \frac{R_0^2(V_\varphi^{(0)})^2}{2R_1^2}\right) - \left(\frac{p_a}{\rho} + \frac{R_0^2(V_\varphi^{(0)})^2}{2R_a^2}\right)\left(\frac{R_1}{R_0}\right)^2\left(\frac{R_i}{R_a}\right)^2\right\}\right]. \tag{A.21}$$

To compare this result with (17), one has to set $V_r = 0$, where $a_2/a_1 = 1$. Furthermore, since (17) was obtained in Lagrange co-moving coordinates, one has to set (except for the log-terms) $R_a = R_1$, and $R_i = R_0$, by which the bracket $\{\dots\} = 0$. One thus has

$$(V_r^{(0)})^2 + (V_\varphi^{(0)})^2\left(1 - \frac{\log(R_a/R_i)}{\log(R_1/R_0)}\right) = 0. \tag{A.22}$$

Putting for a thin tube

$$R_a = R_i + \Delta R_i, \quad \Delta R_i/R_i \ll 1,$$
$$R_1 = R_0 + \Delta R_0, \quad \Delta R_0/R_0 \ll 1, \tag{A.23}$$

one finds that

$$\frac{\log(R_a/R_i)}{\log(R_1/R_0)} \approx \frac{R_0}{R_i}\frac{\Delta R_i}{\Delta R_0}. \tag{A.24}$$

For an incompressible tube one has

$$R_i\,\Delta R_i = R_0\,\Delta R_0 \tag{A.25}$$

and one finds that

$$\frac{\log(R_a/R_i)}{\log(R_1/R_0)} \approx \left(\frac{R_0}{R_i}\right)^2. \tag{A.26}$$

According to (9) and (10) one has $V_\varphi^{(0)} = V_r^{(0)}\sin\beta$, hence for (A.22)

$$\sin^2\beta = \frac{1}{(R_0/R_i)^2 - 1}, \tag{A.27}$$

which for $R_i \ll R_0$ is

$$\sin\beta \approx R_i/R_0 = R_{im}/R_0, \tag{A.28}$$

where R_{im} is the vortex core radius at which $V_r = 0$. Comparing (A.28) with (18), one has to set $r_1 = R_{im}$ and $R_0 = V_0$, which shows that (A.28) is the same as (18).

B. Stability of the Pinch Confined Inside the Imploded Vortex

The density of the destabilizing force acting on the pinch plasma is

$$f_H = H^2/4\pi\lambda, \tag{B.1}$$

where λ is the wavelength of a disturbance by which the pinch plasma is displaced in the radial direction. But by pushing against the wall of the vortex the pinch plasma is then also subject to the buoyancy force

$$f_B = \left(1 - \frac{\rho_p}{\rho}\right)\frac{dp}{dr}, \tag{B.2}$$

where ρ_p is the density of the pinch plasma.

At a maximum implosion one has at the vortex core wall $v_r = 0$, and thus according to (A.12)

$$\frac{dp}{dr} = \rho\left(\frac{\ell^2}{r^3} - \frac{\dot F}{r}\right). \tag{B.3}$$

From (A.14) one has for $R_i = R_{im}$, where $V_r = 0$

$$\dot F = \frac{R_i}{2}\frac{dV_r^2}{dR_i}\bigg|_{R_{im}} = \frac{p_a}{\rho\log(R_{im}/R_a)}, \tag{B.4}$$

hence

$$\frac{dp}{dr} = \rho\left(\frac{\ell^2}{r^3} - \frac{a^2}{r}\right), \tag{B.5}$$

where

$$a = \sqrt{p_a/\rho}\,/\sqrt{\log(R_{im}/R_a)} \tag{B.6}$$

or approximately

$$a \approx V_r^{(0)}. \tag{B.7}$$

The buoyancy force vanishes at the position where $dp/dr = 0$. This is at

$$r = \ell/a = r_1 \tag{B.8}$$

because $\ell = r_0 V_\varphi^{(0)} = r_0 V_r^{(0)}\sin\beta$, $a = V_r^{(0)}$ and $\sin\beta = r_1/r_0$.

Expanding $(1/\rho)\,\mathrm{d}p/\mathrm{d}r$, given by (B.5), into a Taylor series at $r = r_1$, where $\mathrm{d}p/\mathrm{d}r = 0$, one has

$$\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}r} = \frac{1}{\rho}\frac{\mathrm{d}^2 p}{\mathrm{d}r^2}\bigg|_{r_1}(r - r_1) + \ldots = -2\,\frac{a^4}{\ell^2}(r - r_1) = -2\,(V_r^{(0)})^2\,(r - r_1)/r_1^2. \qquad \text{(B.9)}$$

Inserting (B.9) into (B.2) one has

$$f_\mathrm{B} = -2\rho\left(1 - \frac{\rho_\mathrm{p}}{\rho}\right)(V_r^{(0)})^2\,(r - r_1)/r_1^2. \qquad \text{(B.10)}$$

Stability requires that

$$f_\mathrm{H} + f_\mathrm{B} < 0. \qquad \text{(B.11)}$$

With $H/\sqrt{4\pi\rho_\mathrm{p}} = v_\mathrm{A} \approx V_r^{(0)}$ one finds from (B.1), (B.10) and (B.11)

$$\lambda > \frac{1}{2}\left(\frac{r_1^2}{r - r_1}\right)\frac{1}{\rho/\rho_\mathrm{p} - 1}. \qquad \text{(B.12)}$$

For cases of interest $\rho_\mathrm{p} \ll \rho$, and one has

$$\lambda > \frac{1}{2}\left(\frac{r_1^2}{r - r_1}\right)\frac{\rho_\mathrm{p}}{\rho}. \qquad \text{(B.13)}$$

Stability should be assured for all wavelengths down to $\lambda \sim r_1$, requiring that

$$\frac{r - r_1}{r_1} \geq \frac{\rho_\mathrm{p}}{\rho}. \qquad \text{(B.14)}$$

For the example $\rho_\mathrm{p}/\rho \cong 0.1$ one would have $r > 1.1\,r_1$, which shows that a small penetration into the vortex core wall is sufficient to assure stability.

[1] F. Winterberg, Proceedings of Ninth NASA Advanced Space Propulsion Workshop, March 11–13, 1998, Jet Propulsion Laboratory, Pasadena, California.
[2] F. Winterberg, Z. Naturforsch. **53a,** 933 (1998).
[3] M. Tabak, J. Hammer, M. E. Glinsky, W. L. Kruer, S. C. Wilks, J. Woodworth, E. M. Campbell, and M. D. Perry, Physics of Plasmas **1,** 1626 (1994).
[4] F. Winterberg, Z. Physik **A284,** 43 (1978).
[5] G. Birkhoff, D. P. MacDougall, E. M. Pugh, and G. Taylor, Appl. Physics **19,** 563 (1948).
[6] F. Winterberg, Atomkernenergie **28,** 123 (1976).
[7] F. Winterberg, Nature London **356,** 298 (1975).
[8] M. D. Perry and G. Mourou, Science **264,** 917 (1994).
[9] F. Winterberg, The Physical Principles of Thermonuclear Explosive Devices, Fusion Energy Foundation, New York 1981.